Teaching the Costs of Uncoordinated Supply Chains

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Supply-chain management has become a prominent area for teaching and research. Academics and managers realize that communication and coordination among members of a supply chain enhance its effectiveness, creating financial benefits to be shared by the members. We have collected numerical examples covering (1) location decisions, (2) centralized warehousing, (3) lot sizing with deterministic demand, (4) demand forecasting, (5) pricing, and (6) lot sizing with stochastic demand in a newsvendor environment. The examples are suitable for classroom use, and they illuminate the rewards supply-chain members can obtain by eliminating naturally occurring supply-chain inefficiencies and the costs of not doing so. (*Professional: OR/MS education. Supply-chain management.*)

When each member of a group tries to maximize his or her own benefit without regard to the impact on other members of the group, the overall effectiveness of the group may suffer. Such inefficiencies often creep in when rational members of supply chains optimize individually instead of coordinating their efforts. Nowadays, companies should not act in isolation, as success in the global marketplace requires whole supply chains to compete against other supply chains (Davis 1994). Supply-chain members must recognize the natural inefficiencies that may develop and work to eliminate them, so that the supply chain as a whole can compete effectively.

Real-world examples of supply-chain coordination abound (Lee and Ng 1998, Munson et al. 1999). More concretely, students of business can work through numerical examples to better understand and appreciate from a theoretical perspective the simple but powerful concept of supply-chain coordination and its benefits. The examples we present span issues of location, warehousing, inventory, information sharing, and pricing. They are generally simplified versions of ideas that can be found in the literature tailored for classroom use. We introduce each example with a nontechnical discussion of the experiences of Isaac's Ice Cream, a fictitious sole proprietorship.

Examples 1 and 2 concern *horizontal coordination*, that is, coordination among entities on the same level of the supply chain. Examples 3 through 6 describe *vertical coordination*, that is, coordination among entities on different levels of the supply chain (for example, a retailer and its supplier).

Example 1: Location Decisions

Isaac's Ice Cream had been selling very well in the city, but Isaac wished to expand his market to reach summertime tourists by selling his ice cream from small

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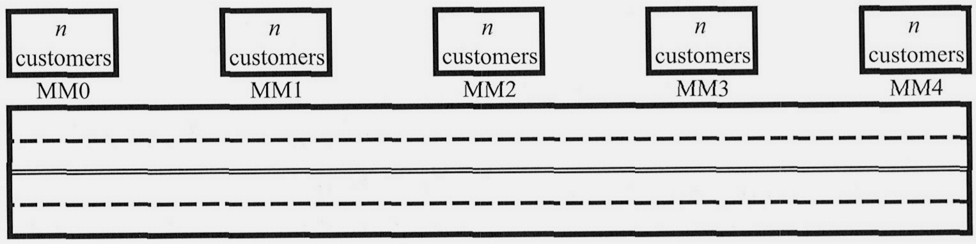

Figure 1: In a four-mile stretch of road with five mile markers, ($MMi, i = 0, \ldots, 4$), the same number of customers, n, cluster around each of the five mile markers. The franchise must decide where along the road to locate two franchisees.

carts along the boardwalk on the beach. He offered these "franchises" to two young entrepreneurs, Sally and Pete. Isaac obtained permits allowing both carts to locate anywhere along the four-mile boardwalk. Moving these rolling stores to new locations entailed essentially no setup cost. Believing that Sally and Pete would know best where to locate because they were close to the customers, Isaac allowed them to locate anywhere they wished, suggesting only that they stay out of each other's way. On the first day, Sally took a cart and told Pete that she would cover the north end of the boardwalk and he could have the south end. Pete agreed, and they went their separate ways.

Sally parked her cart about one mile from the north end of the boardwalk. Morning sales were steady; however, she noticed that the only buyers were those walking from the north end, while quite a few strollers walking past from the south already had ice cream. So, Sally walked a few hundred yards south and, to her dismay, noticed that Pete's cart was right there, almost three full miles from the south end. Infuriated, Sally snuck around Pete's back and set up her cart in a new location about 300 yards south of Pete. Similar maneuvering continued back and forth all day. At 5:00, Isaac found Sally and Pete's carts right across from each other halfway down the four-mile boardwalk. The two were covered with ice cream, apparently from an altercation. People strolled by, refreshments from nearby concession stands in hand. What happened?

A franchise has multiple outlets to serve customers, spread out over a town, a state, a country, or even multiple continents. To maximize market coverage, franchisors generally strive for many locations, even if their market areas overlap (Marsh 1992). However, the franchisees who own the individual outlets generally want to maximize their market access, and they do not want their service areas cannibalized by another franchisee. Therefore, the franchisor may sometimes need to control the allocation of territories served or the locations of the franchisees. In one case, KFC tried to appease franchise owners by offering a pass-through royalty equal to two percent of the sales made by new outlets opening near them (Ruffenach 1992). Sally and Pete's conflict illustrates the detrimental effects to individual franchisees and the entire franchise of letting franchisees choose their own locations. Game theory texts (for example, Rasmusen 1989) include more general Hotelling models (games).

Suppose that a franchisor wishes to open fast-food restaurants along a stretch of road four miles long. Potential customers cluster along mile markers (MM) 0, 1, 2, 3, and 4, with n customers in each cluster (Figure 1). Customer demand is sensitive primarily to distance traveled. Specifically, for each customer, $D = a(b - d)$, where D = weekly demand, a is a constant > 0, b is a constant > 4, and d is the distance traveled in miles. Both the franchisor and the franchisees wish to maximize weekly demand.

Case 1: Two Franchisees Whose Locations Are Coordinated by the Franchisor

If the franchisor can locate the two franchisees anywhere along the four-mile stretch of road, total demand for the entire franchise will be maximized when the first franchise (F1) is located at mile marker 1 (MM1) and the second (F2) is located at mile marker 3 (MM3). (We can easily verify this result by enumerating the possible scenarios.)

The total distance customers travel will be $3n$ miles, and total franchise demand will equal $na(b-1)+na(b-0)+na(b-1)+na(b-0)+na(b-1)=na(5b-3)$. As we might expect, this demand exceeds that obtainable by only one franchisee. In the single-franchisee case, simple enumeration shows that the location of the single franchisee should be at MM2, and the franchise demand will equal $na(b-2)+na(b-1)+na(b-0)+na(b-1)+na(b-2)=na(5b-6)$.

Case 2: Two Franchisees That Control Their Own Locations

In this case, F1 and F2 act in their own interest to maximize their own demand, knowing that the other franchisee exists and then reacting accordingly. Without loss of generality, assume that F1 chooses its location first. To maximize its own demand, it will locate at MM2. F2 then has two choices: (1) to also locate at MM2, or (2) to locate somewhere other than MM2. If F2 also locates at MM2, the two franchisees will share the demand of $na(5b-6)$, and each will end up with half of it. On the other hand, F2 could capture the entire demand from two other locations, say MM0 and MM1, by locating somewhere between them (say MM0 + 0.5). In that case, F2's total demand would be $na(b-0.5)+na(b-0.5)=na(2b-1)$, which is less than its MM2 location demand of $na(5b-6)/2=na[(5/2)b-3]=na[2b+(b/2-3)]>na(2b-1)$, since $b>4$.

Therefore, assuming that demand is primarily a function of distance, two rational franchisees choosing their locations simultaneously to maximize their own profits will both locate at the midpoint of the stretch of road, sharing the same total franchise demand, $na(5b-6)$, that one franchisee alone would have had. On the other hand, either through contractual agreement or through the franchisor's direction and coordination, the two franchisees can cooperate and locate at MM1 and MM3 (Case 1). Both will then experience greater demand, and the total franchise will receive a demand of $na(5b-3)$.

Example 2: Centralized Warehousing

Over time, Isaac's Ice Cream has grown and now sells certain products through 200 company-owned retail outlets split equally between two states. In both states, Isaac leases warehouse space for storage of goods strictly by the square foot. In the firm's first state of operation, it leased warehouse space near each shop. However, when Isaac expanded to the second state he tried storing goods for all 100 shops in that state at a central location. Although transportation costs and lead times are somewhat higher in the second state, Isaac is puzzled when he reviews his books because the second state performs much better on certain other criteria.

While each of the 100 warehouses in the original state stores fewer goods and has fewer orders to fill than the centralized warehouse in the second state does, the sum of the individual warehouse costs is much larger. In fact, the total warehousing costs are 90 percent lower in the second state. Isaac has heard of economies of scale, but this result surprises him because he is not paying any fixed land or building costs at the warehouses; he pays only for storage space and ordering and receiving costs. In addition, the firm has always carried safety stock to protect against unusually high demand. For consistency, Isaac keeps the same amount of total system safety stock in both states. To his surprise, stores in the original state receive 70 percent service, while stores in the new state receive (essentially) 100 percent service. "How can this be?" Isaac wonders. "How can centralization dramatically decrease my costs while dramatically increasing my service level?"

In this example, we consider two benefits of centralized warehousing: (1) economies of scale in setup and holding costs, and (2) risk pooling in a stochastic-demand environment. Centralized warehousing can be implemented by single companies for their field sites, franchisors for their franchisees, or even suppliers for their competing customers.

Economic Order Quantity Costs

The economic-order-quantity (EOQ) model nicely illustrates the economies-of-scale benefits of centralization. For simplicity, assume that each client (retailer or franchisee) has the same holding cost H, setup cost S, and annual demand D. Further assume that the supplier's (or central warehouser's) holding and setup costs are also H and S, respectively. The EOQ cost for each client warehousing on its own is $\sqrt{2DSH}$. For N clients, the total EOQ costs for that level of the supply chain are $N\sqrt{2DSH}$. On the other hand, if the supplier combines the demands of every client and warehouses the items centrally, then the total EOQ costs are $\sqrt{2(ND)SH}$.

Therefore, the savings percent for the channel obtainable from centralized warehousing is

$$\frac{(N-\sqrt{N})\sqrt{2SDH}}{N\sqrt{2SDH}} = 1 - \frac{\sqrt{N}}{N}.$$

Thus, with only four client sites, the channel saves 50 percent on holding and setup costs. The amount rises to 80 percent for 25 sites and 90 percent for 100 sites. (Obtaining these savings may require additional costs for centralization, such as increased transportation.)

Risk Pooling—Newsvendor Environment

Eppen (1979) illustrates the risk-pooling benefits ("statistical economies of scale") of centralized warehousing in a one-period newsvendor environment with normal probability distributions. Evans (1997) and many other authors of operations management textbooks describe how to determine the optimal order quantity in this environment to minimize expected overage and underage costs. Eppen (1979) shows that firm i choosing its optimal order quantity has expected overage and underage costs equal to $K\sigma_i$, where σ_i is firm i's standard deviation of demand. (Eppen gives the value of K, but it is not needed for classroom use of this example.)

If we assume that each client has the same overage and underage costs per unit, and the same, but independent, normal probability demand distribution with mean μ and variance σ^2, then each client warehousing on its own has total expected overage and underage costs of $K\sigma$. For N clients, the total expected overage and underage costs for that level of the supply chain are $NK\sigma$. On the other hand, if the supplier combines the demands of its clients and warehouses the items centrally, the demand distribution for all clients combined is also normal with mean $N\mu$ and variance $N\sigma^2$, and the total expected overage and underage costs are $K\sqrt{N\sigma^2} = \sqrt{N}K\sigma$. Just as in the EOQ example, the savings percent equals $1-\sqrt{N}/N$.

Risk Pooling—Safety Stocks and Service Levels

Students of business typically learn how to compute safety stocks under continuous-review and periodic-review inventory systems with normally distributed demands (Krajewski and Ritzman 2002). Specifically, the safety stock equals $z\sigma$, where z represents the number of standard deviations above the mean needed to achieve a desired cycle service level and σ is the standard deviation of demand over the protection interval. By using centralization, the supplier (1) can decrease the total system safety stock, or (2) can increase service levels using the same total system safety stock.

If we assume that each of N clients has the same, but independent, normal probability demand distribution with mean μ and variance σ^2, then the total amount of safety stock for that level of the supply chain is $Nz\sigma$. On the other hand, if the supplier combines the demands of all the clients and warehouses the items centrally, the demand distribution for all clients combined is also normal with mean $N\mu$ and variance $N\sigma^2$, and the total amount of safety stock is $z\sqrt{N\sigma^2} = \sqrt{N}z\sigma$. As in the EOQ example, the savings percent equals $1-\sqrt{N}/N$.

If instead the centralized warehouser keeps the total safety stock the same as the clients did warehousing on their own, the new higher z-value can be imputed as follows: $Nz_{old}\sigma = \sqrt{N}z_{new}\sigma$, or $z_{new} = \sqrt{N}z_{old}$. By coordinating just a few clients, a supply chain can attain a much higher level of service with the same amount of safety stock (Table 1).

The percent of cost savings from centralizing safety stocks varies with the number of clients (Table 2). These savings are applicable to all three situations:

Number of Clients (N)	Cycle Service Level 70.00% ($z_{old} = 0.5244$)	80.00% ($z_{old} = 0.8416$)	90.00% ($z_{old} = 1.2816$)
2	77.08%	88.30%	96.50%
3	81.81%	92.75%	98.68%
4	85.29%	95.38%	99.48%
5	87.95%	97.01%	99.79%
6	90.05%	98.04%	99.92%
7	91.73%	98.70%	99.97%
8	93.10%	99.14%	99.99%
9	94.22%	99.42%	99.99%
10	95.14%	99.61%	100.00%
11	95.90%	99.74%	100.00%
12	96.54%	99.82%	100.00%
13	97.07%	99.88%	100.00%
14	97.51%	99.92%	100.00%
15	97.89%	99.94%	100.00%
25	99.56%	100.00%	100.00%
50	99.99%	100.00%	100.00%
100	100.00%	100.00%	100.00%

Table 1: This table displays the cycle service levels obtained from centralizing decentralized service levels of 70, 80, and 90 percent. For the same level of safety stock, centralized warehousing provides an increased cycle service level according to the formula $z_{new} = \sqrt{N} z_{old}$.

Number of Clients (N)	Cost Savings (%)
2	29.29
3	42.26
4	50.00
5	55.28
6	59.18
7	62.20
8	64.64
9	66.67
10	68.38
11	69.85
12	71.13
13	72.26
14	73.27
15	74.18
25	80.00
50	85.86
100	90.00
1,000	96.84

Table 2: With regard to (1) EOQ costs, (2) newsvendor model costs, or (3) safety-stock costs under continuous or periodic review systems, centralized warehousing reduces those costs by a percentage equal to $1 - \sqrt{N}/N$.

(1) EOQ costs, (2) newsvendor model costs, and (3) risk pooling of safety stocks. Clearly, coordinating just a few clients can produce significant savings.

Example 3: Coordinated Lot Sizes with Deterministic Demand

Isaac's Ice Cream produces 1 million boxes of a special frozen treat per year exclusively for a large grocery chain. The chain has been ordering 8,165 boxes at a time (presumably its EOQ) approximately every three days. In college, Isaac heard that manufacturers should produce in an integer multiple of demand when orders are lumpy. Because this product has an expensive setup cost and a very fast production rate, Isaac has found it cheapest to produce 48,990 boxes at a time (every 18 days).

This lumpy demand seems beneficial; Isaac notices that his total annual setup and holding costs amount to \$91,856, whereas the EOQ model tells him that his costs would be \$100,000 if demand were not lumpy. That revelation makes him wonder, "Is my incoming demand lumpy enough? If my customer ordered larger amounts less frequently, would I save even more money?" Realizing that his firm could also produce about 49,000 units at a time by making four times incoming orders of 12,250 units, Isaac computes the costs and learns that he could save \$4,086 by doing so. He wonders whether passing some of the savings along to the grocery chain in the form of a quantity discount would induce the chain to increase its order size accordingly.

Most students learn about the EOQ model and possibly some of its extensions, such as the EOQ with finite production rate or the EOQ with all-units quantity discounts (Krajewski and Ritzman 2002). However, they seldom explore the effect that those lumpy orders of size Q^* have on the suppliers. While optimal for a retailer acting alone, the EOQ is seldom optimal for a supply chain consisting of the retailer and its supplier. Based on this realization, Hewlett-Packard uses a mathematical program to determine inventory levels for some products at both its distribution centers and dealer stores, thereby minimizing systemwide inventory levels (Lee and Billington 1995).

In this example, we assume that a single retailer operates under the typical EOQ assumptions, and it purchases its product from a single supplier (with an essentially infinite production rate). Under these conditions, it is optimal for the supplier's lot size to be an integer multiple, n, of the retailer's lot size (Lee and Rosenblatt 1986).

Notation

D = annual demand.
S_s = supplier's setup cost.
S_r = retailer's setup cost.
H_s = supplier's annual holding cost per unit.
H_r = retailer's annual holding cost per unit.
Q = retailer's order size.
n = supplier's integer lot-size multiplier.
nQ = supplier's lot size.
$\lfloor x \rfloor$ = the greatest integer $\leq x$.

From Munson and Rosenblatt (2001), we can derive the following formulas. Total annual supply-chain holding and setup costs are equal to

$$TC = \left(\frac{D}{nQ}\right) S_s + \left[\frac{(n-1)Q}{2}\right] H_s + \left(\frac{D}{Q}\right) S_r + \left(\frac{Q}{2}\right) H_r.$$

The first two terms represent the supplier's annual setup and holding costs, respectively, and the second two terms represent the retailer's annual setup and holding costs, respectively.

When the parties optimize independently, the retailer orders Q^* and the supplier orders n^*Q^*, where

$$Q^* = \sqrt{\frac{2DS_r}{H_r}}$$

and

$$n^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \frac{8DS_s}{H_s(Q^*)^2}}\right) \right\rfloor.$$

When they optimize jointly, they go through three steps:

Step 1: Compute

$$n^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + \text{Max}\left\{0, \frac{4S_s(H_r - H_s)}{S_r H_s}\right\}}\right) \right\rfloor.$$

Step 2: Compute

$$\overline{S} = S_s/n^* + S_r \quad \text{and} \quad \overline{H} = (n^* - 1)H_s + H_r.$$

Step 3: Compute

$$Q^* = \sqrt{2D\overline{S}/\overline{H}}.$$

The terms $\overline{S}$ and $\overline{H}$ represent the system setup cost per retailer's order and the annual holding cost per unit for the system, respectively. The retailer orders Q^* and the supplier orders n^*Q^*. At those quantities, the total system setup and holding cost equals $TC^* = \sqrt{2D\overline{S}\,\overline{H}}$.

For example, consider a product with an annual demand of 25,000 units, $S_s = \$200$, $S_r = \$40.50$, $H_s = \$2.00$, and $H_r = \$2.50$.

If the firms act independently, the retailer will order its EOQ of 900 units, and n^* will equal 3, implying that the supplier's lot size will be 3(900) = 2,700 units. The total system cost of these lot sizes equals \$5,902, and the retailer's portion is \$2,250.

On the other hand, if the firms optimize jointly, $n^* = 1$, $\overline{S} = \$240.50$, $\overline{H} = \$2.50$, and $Q^* = 2{,}193$ units. Thus, the retailer orders 2,193 units and so does the supplier (1(2,193)). The total system cost using these values is \$5,483, which is 7.1 percent lower than the cost when the firms do not coordinate.

The supplier's costs decrease with joint optimization by \$3,652 − \$2,280 = \$1,372. However, the retailer's costs increase (because it no longer orders its EOQ) by \$3,203 − \$2,250 = \$953. Therefore, some of the supplier's savings should be redistributed in compensation to the retailer. A quantity discount for ordering 2,193 units instead of 900 units is an excellent way to entice the retailer to agree to this change in policy. Monahan (1984), Lee and Rosenblatt (1986), and Weng and Wong (1993) present generalized versions of this problem. Munson and Rosenblatt (2001) show that the savings continue to grow when the supply chain is expanded to three levels by including the supplier's supplier in the model.

S_s/S_r	Q^*_{new}/Q^*_{old}	Cost Savings from Coordination (%)
1	1.41	5.72
2	1.73	13.40
3	2.00	20.00
4	2.24	25.46
5	2.45	30.01
6	2.65	33.86
7	2.83	37.15
8	3.00	40.00
9	3.16	42.50
10	3.32	44.72
15	4.00	52.94
20	4.58	58.34
50	7.14	72.53
100	10.05	80.29

Table 3: This table displays the benefits from lot-sizing coordination between a retailer and its supplier when the supplier uses a lot-for-lot production policy. The retailer should increase its order size by a factor of $\sqrt{S_s/S_r+1}$ (Monahan 1984). The system percentage cost savings from coordination equals $1-[(2\sqrt{S_s/S_r+1})/(2+S_s/S_r)]$.

In the special case in which the supplier always utilizes a lot-for-lot production policy ($n^*=1$), the benefits of coordination increase as the ratio of the supplier's setup cost to the retailer's setup cost increases (Table 3). Other things being equal, it is more important to the supply chain for the retailer to increase its order size when a lot-for-lot supplier has a large setup cost.

Example 4: Coordinated Demand Forecasting

Isaac's Ice Cream sells certain ice cream sandwiches to regional food wholesalers who distribute them through local grocery stores. Isaac's sister Janet supplies most of the primary ingredients. One day Janet and Isaac met for their annual review. The wholesalers had been complaining to Isaac about late deliveries, yet eight times during the last year he was forced to purchase extra storage space for finished goods because he had run out of room at the factory. He had been late with many deliveries because the supplies from Janet had arrived late to him.

"It's not my fault," Janet exclaimed. "I'm running a small company. I do have other customers, and I can't just drop everything to fill your orders. We go for weeks at a time hearing nothing from you, and then all of a sudden you place an order for three months' worth of demand. I don't get it. One of your grocery retailers gave me data on her sales of your ice cream sandwiches last month. Other than a few spikes on weekends, her sales have been very steady. Yet I never know what to expect from you. I can't afford to hold inventory for you for months at a time. Do you have steady sales at all of the grocery outlets? If so, why do I get these crazy orders from you? I never know what to expect! We're family. Talk to me!"

Many business school students get the opportunity to play the beer game during their college careers (Sterman 1989, 1992). In this popular role-playing game, students act out the roles of a retailer, wholesaler, distributor, or manufacturer in a supply chain who are determining order sizes in attempts to minimize back-order and inventory-holding costs. Although players are rewarded based on the total costs of their team, they invariably play the game by focusing on minimizing their own costs independently. An important feature of the game is that the members of the supply chain can communicate only through the orders they place, that is, only the retailer sees the final consumer demand.

Typically the game results in wide oscillations in inventory, back orders, and order sizes, which are most pronounced for the upstream players, that is, for the distributor and especially for the manufacturer. Procter and Gamble executives have coined the term *bullwhip effect* to describe this phenomenon in their firm's supply chain. Lee et al. (1997b) have identified four major sources of the bullwhip effect that are consistent with rational managerial behavior: (1) demand forecast updating, (2) order batching, (3) price fluctuation, and (4) rationing and shortage gaming. Kaminsky and Simchi-Levi (1998) report the results of a computerized beer game that allows for easy manipulation of some of the game's parameters. They have successfully used the game in classroom settings.

Examples abound of high-profile manufacturers suffering the impacts of poor forecasting. For instance, in the mid-1990s, IBM and Apple Computers made forecast errors that caused them huge losses and eroded their market shares. The effects rippled up and down their supply chains (Fisher et al. 1997). To improve forecasting, some firms have strived to increase information sharing throughout their supply chains. For example, JCPenney has reported almost daily communications for one of its clothing lines back through the manufacturer (Robinson Manufacturing), the textile mill (Milliken and Company), and even the fiber producer (Du Pont) (Thornton 1995).

Using a simplified beer game setting with no player discretion, we can unambiguously demonstrate the benefits of coordinating demand forecasts. Lee et al. (1997a, b) suggest sharing information about consumer demand with all supply-chain members or having one member perform forecasting for all the members. Students can gain an understanding of the potential power of this strategy by building two spreadsheet models, one with forecasting by the individual players and the other with all players using the retailer's forecasts.

Our model illustrates a two-firm channel using simple (naive) forecasting and one-period lead times. It can be extended to include more channel members, more complicated forecasting schemes, or longer lead times, as desired by the instructor. We simulate a 20-period game with a retailer receiving orders from consumers and placing orders with its wholesaler. The wholesaler has unlimited production capacity. A one-period lead time applies to all orders. We use *naive* forecasts, that is, we use this period's demand as the next period's forecast. Both players use an order-up-to policy, in which the order size equals next period's forecast minus the inventory position, which includes current inventory plus scheduled receipts minus back orders. In each period, the retailer moves first and the

	A	B	C	D	E	F	G	H	I	J	K	L
1				Retailer						Wholesaler		
2			Next					Next				
3		Consumers'	Period's	On-Hand		Order	In-Transit	Period's	On-Hand		Order	In-Transit
4	Period	Orders	Forecast	Inventory	Back Orders	Placed	Inventory	Forecast	Inventory	Back Orders	Placed	Inventory
5	0			5	0		5		5	0		5
6	1	5	5	5	0	0	0	0	10	0	0	0
7	2	5	5	0	0	5	5	5	5	0	0	0
8	3	5	5	0	0	5	5	5	0	0	5	5
9	4	5	5	0	0	5	5	5	0	0	5	5
10	5	5	5	0	0	5	5	5	0	0	5	5
11	6	20	20	0	15	35	5	35	0	30	65	65
12	7	20	20	0	30	20	50	20	15	0	5	5
13	8	20	20	0	0	20	20	20	0	0	20	20
14	9	20	20	0	0	20	20	20	0	0	20	20
15	10	20	20	0	0	20	20	20	0	0	20	20
16	11	50	50	0	30	80	20	80	0	60	140	140
17	12	30	30	0	40	10	70	10	70	0	0	0
18	13	30	30	0	0	30	30	30	40	0	0	0
19	14	30	30	0	0	30	30	30	10	0	20	20
20	15	30	30	0	0	30	30	30	0	0	30	30
21	16	10	10	20	0	0	0	0	30	0	0	0
22	17	10	10	10	0	0	0	0	30	0	0	0
23	18	50	50	0	40	90	30	90	0	60	150	150
24	19	10	10	0	20	0	60	0	90	0	0	0
25	20	10	10	30	0	0	0	0	90	0	0	0
26	Total			70	175		410		395	150		490

Figure 2: The Excel Microsoft simulation of a simplified beer game displays inventory and back orders for a two-firm supply chain with forecasting based on each party's own demand. We provide formulas in Table 4.

	A	B	C	D	E	F	G	H	I	J	K	L
31				Retailer						Wholesaler		
32			Next					Next				
33		Consumers'	Period's	On-Hand		Order	In-Transit	Period's	On-Hand		Order	In-Transit
34	Period	Orders	Forecast	Inventory	Back Orders	Placed	Inventory	Forecast	Inventory	Back Orders	Placed	Inventory
35	0			5	0		5		5	0		5
36	1	5	5	5	0	0	0	5	10	0	0	0
37	2	5	5	0	0	5	5	5	5	0	0	0
38	3	5	5	0	0	5	5	5	0	0	5	5
39	4	5	5	0	0	5	5	5	0	0	5	5
40	5	5	5	0	0	5	5	5	0	0	5	5
41	6	20	20	0	15	35	5	20	0	30	50	50
42	7	20	20	0	30	20	50	20	0	0	20	20
43	8	20	20	0	0	20	20	20	0	0	20	20
44	9	20	20	0	0	20	20	20	0	0	20	20
45	10	20	20	0	0	20	20	20	0	0	20	20
46	11	50	50	0	30	80	20	50	0	60	110	110
47	12	30	30	0	40	10	70	30	40	0	0	0
48	13	30	30	0	0	30	30	30	10	0	20	20
49	14	30	30	0	0	30	30	30	0	0	30	30
50	15	30	30	0	0	30	30	30	0	0	30	30
51	16	10	10	20	0	0	0	10	30	0	0	0
52	17	10	10	10	0	0	0	10	30	0	0	0
53	18	50	50	0	40	90	30	50	0	60	110	110
54	19	10	10	0	20	0	60	10	50	0	0	0
55	20	10	10	30	0	0	0	10	50	0	0	0
56	Total			70	175		410		230	150		450

Figure 3: This Microsoft Excel simulation of a simplified beer game displays inventory and back orders for a two-firm supply chain with forecasting for both parties based on the actual consumers' orders. We provide formulas in Table 4. Compared to Figure 2 (uncoordinated forecasting), the wholesaler's total on-hand inventory is 42 percent smaller when demand information is shared.

wholesaler follows. Each player first moves in-transit inventory, then fills back orders and new orders to the extent possible, and then places new orders (to be received in the succeeding period, subject to availability). Both players begin with five units in inventory and five units in transit (to be received in period 1).

This type of simulation can be shown in class or given as a computer assignment for students. Figures 2 and 3 show the Microsoft Excel spreadsheets used in our example. Table 4 presents the applicable formulas. In Figure 2, the wholesaler's forecast equals the order received from the retailer in that period. In Figure 3, the wholesaler's forecast equals the retailer's forecast, which equals the consumer orders in that period. Figure 3 represents sharing of demand or forecasting information between the parties. For this example, when demand information is shared, the wholesaler's total on-hand inventory held over 20 periods is 165 units (42 percent) smaller than the case of no information sharing. In the uncoordinated case (Figure 2), the wholesaler is overreacting to the retailer's catch-up orders by assuming that future underlying consumer demand will be larger than it actually turns out to be. Lee et al. (1997a, b) provide real-world examples of successful information sharing among supply-chain members.

Example 5: Coordinated Pricing

Each year Isaac's Ice Cream sells its special blend at the state fair. The fair lasts for only a few days, and

Cell	Formula	Explanation
C6	= B6	Retailer's forecast equals this period's consumer demand.
D6	= MAX(D5 + G5 − E5 − B6, 0)	Retailer's ending inventory after this period.
E6	= MAX(E5 + B6 − D5 − G5, 0)	Retailer's ending back orders after this period.
F6	= MAX(C6 − (D6 + J5 − E6), 0)	Order-up-to policy = next period's forecast − inventory position.
G6	= MIN(F6 + J5, I5 + L5)	Amount put into shipment from the wholesaler this period.
H6	= F6	Wholesaler's forecast equals retailer's order size this period.
I6	= MAX(I5 + L5 − J5 − F6, 0)	Wholesaler's ending inventory after this period.
J6	= MAX(J5 + F6 − I5 − L5, 0)	Wholesaler's ending back orders after this period.
K6	= MAX(H6 − (I6 − J6), 0)	Order-up-to policy = next period's forecast − inventory position.
L6	= K6	Assuming the wholesaler has infinite production capacity.
Change for Figure 3		
H36	= B36	Wholesaler's forecast equals this period's consumer demand.

Table 4: These are the Microsoft Excel formulas for the forecasting simulation shown in Figures 2 and 3.

Isaac sells this particular item only at this annual fair. Thus, he must determine ahead of time the appropriate quantity to produce and deliver. He sells the ice cream through an independently operated booth at the fair that sells many other food items. Isaac has been successfully selling his special blend of ice cream for a number of years, and, by monitoring the price the booth charges consumers, he has determined that demand is very price sensitive.

Last year Isaac charged his retailer \$3.00 per pint, and it cost him \$1.00 per pint to produce. The retailer charged \$4.00 per pint and sold 2,000 units. The retailer's variable costs consisted primarily of the wholesale price paid to Isaac. Isaac's research indicated that he could double sales to 4,000 units if the retailer reduced the price to \$3.00. The math seemed simple to Isaac: "This year I'll lower the wholesale price to \$2.50 and tell my retailer to sell the ice cream for \$3.00 per pint. I'll earn \$6,000 instead of \$4,000, and the retailer will still earn \$2,000, so he will be no worse off." However, the retailer ignored Isaac's suggestion and only lowered the price to \$3.75, inducing a demand of 2,500 units. Compared to last year, Isaac's profits fell from \$4,000 to \$3,750, but the retailer's profits rose from \$2,000 to \$3,125. What happened?

Students taking any introductory microeconomics class learn that a monopolist will maximize profits by following the *golden rule of output determination*, that is, by selecting the output level at which marginal revenue equals marginal cost (Mansfield 1983). But what happens if a retailer and its supplier are both monopolists and part of the retailer's marginal cost is the wholesale price? The supply chain loses money when the firms do not coordinate their pricing but instead rely on the traditional, sequential method in which the supplier first sets the wholesale price and the retailer reacts accordingly, as shown in Example 5. This example is most appropriate for goods that cannot be stored for long, that is, goods that are perishable or have short life cycles. The computer industry represents such an environment with short and price-sensitive demand. Some computer firms have suffered losses in recent years because of their poor pricing and forecasting practices (Weng 1999).

Case 1: A System with One Retailer and One Supplier

Let the retailer's demand curve be $P = 900 - 2Q$ (where P is the retail price and Q is the quantity sold), and let the marginal costs (exclusive of wholesale price) be \$10 and \$90 for the retailer and supplier, respectively. Total revenue for the retailer is $P \times Q = 900Q - 2Q^2$. Marginal revenue is the derivative of total revenue with respect to Q, which equals $900 - 4Q$. If the firms are considered as one organization, then the optimal quantity Q^* solves marginal revenue = marginal cost: $900 - 4Q = 100$, or $Q^* = 200$. A \$500 price induces a demand of 200 units, so the total channel profits are 200[\$500 – (\$10 + \$90)] = \$80,000.

Next, consider independent optimization. The supplier knows that the retailer will set marginal revenue equal to marginal cost, that is, $900 - 4Q = 10 + W$, where W is the wholesale price charged to the retailer. So, the supplier faces a demand curve from the retailer of $W = 890 - 4Q$. With this linear demand function, the supplier's total revenue is $W \times Q = 890Q - 4Q^2$, and the marginal revenue should equal the supplier's marginal cost, that is, $890 - 8Q = 90$, implying that $Q^* = 100$. After plugging Q^* into the supplier's demand function, the profit-maximizing wholesale price becomes $W^* = 890 - 4(100) = \$490$. Of course, with a wholesale price of \$490, the retailer will also maximize profits by selling 100 units, which will be induced by a retail price of $P^* = 900 - 2(100) = \$700$. With these values, the supplier's profit equals 100(\$490 – \$90) = \$40,000, which is one-half of the amount achievable through cooperative optimization. Furthermore, the retailer's profit equals 100[\$700 – (\$10 + \$490)] = \$20,000, which is one-fourth of the amount achievable through cooperative optimization. Total channel profits are \$40,000 + \$20,000 = \$60,000. Cooperative optimization produces \$20,000 (33 percent) more than independent optimization would produce. (It can be shown (Appendix) that the 33 percent profit increase holds for any linear demand function and associated marginal costs.)

In class, it is also interesting to see if students can determine ways to achieve the desired cooperation between the retailer and the wholesaler. The goal is to get the retailer to sell 200 units by setting a retail price of \$500. However, the retailer will not comply as long as the wholesale price remains \$490. The actual cooperation mechanism used will depend on the relationship between the two firms and their relative power. Students may come up with such ideas as the supplier imposing a retail price of \$500 or a minimum order quantity of 200 units. In addition, either firm could vertically integrate to eliminate the problem.

A quantity discount approach represents an excellent coordination mechanism. If the retailer actually did lower its price to \$500, then the retailer's profit would become \$0, but the supplier's profit would double to \$80,000. Now there are \$20,000 of new channel profits to share between the two parties. Any all-units quantity discount between \$100 and \$200 per unit for orders of size 200 will create \$20,000 of new wealth for the channel, and neither firm will be worse off than with no discount. For example, a discount of \$100 per unit would allocate all of the new profits to the supplier, a discount of \$150 per unit would evenly split the increased profits, or a discount of \$200 per unit would allocate all of the new profits to the retailer. (Jeuland and Shugan 1983 provide a generalized version of this problem.)

Students may wonder why the supplier does not simply lower W to the point where 200 units maximizes the retailer's profit. To find that W, set the retailer's marginal revenue equal to its marginal cost, that is, $900 - 4(200) = 10 + W$, or $W^* = \$90$. At this wholesale price, total channel profits are indeed \$80,000, but the retailer captures all of it while the supplier's profit equals \$0. Thus, while lowering W always helps the total channel, it always hurts the supplier, who would likely be unwilling to comply unless the retailer somehow transferred some money back. A quantity discount avoids such complications.

Case 2: A System with One Retailer and $N-1$ Supplier Tiers

Consider a supply chain with one retailer and $N-1$ supplier tiers. For example, a supply chain consisting of a retailer, the retailer's supplier, and the retailer's supplier's supplier would contain two supplier tiers. If the retailer has a linear demand curve of the form $P = a - bQ$ $(a, b > 0)$, the system percentage profit increase from coordinated pricing vs. individual optimization is

$$\frac{2^{2N-2} - 2^N + 1}{2^N - 1}.$$

(The proof is in the Appendix.) Substantial potential benefits to system profits from coordinating pricing are particularly prevalent in supply chains with multiple tiers (Table 5).

Example 6: Coordinated Newsvendor Lot Sizes

Isaac's Ice Cream sells "homemade" vanilla shakes daily at Sunnyside Park during the summertime. The

Number of Supplier Tiers ($N-1$)	Profit Increase %
1	33.33
2	128.57
3	326.67
4	725.81
5	1,525.40
6	3,125.20

Table 5: This table displays the benefits from coordinated pricing between a retailer and its $N-1$ tiers of single suppliers when the retailer has a linear demand curve of the form $P = a - bQ$ $(a, b > 0)$. The system percentage profit increase equals $(2^{2N-2} - 2^N + 1)/(2^N - 1)$.

pint-sized shakes are sold by the driver of an ice cream truck who stores the mixture in a cooler that keeps it frozen for only a few hours. After that, the driver can sell any remaining (melted) mixture to Pete's Pig Farm for 87.5 cents per pint. The truck driver places her order at night and makes one trip to Isaac's factory the next morning on her way to the park. Isaac produces exactly the amount ordered. The driver pays Isaac \$2.00 per pint, and she sells it in shake form for \$4.00 per pint. (Her other marginal costs for these shakes are minimal.) Daily demand seems to vary fairly evenly (that is, with no noticeable mode) between 50 and 250 units. Isaac produces the mixture at a cost of \$1.00 per pint.

Home for the summer from business college, the truck driver's daughter tells her that the best possible amount for her to order every day is 178 pints. However, Isaac's son attends the same business college, and he is convinced that the truck driver should be trying to sell 242 units per day. Obviously, Isaac's expected profits would rise, but that order size seems very risky for the truck driver, so her expected profits would likely fall. How can Isaac convince the truck driver to order so many more units? And if he compensates her for the greater risk, will any excess profits remain for him?

In Example 2, we explored the risk-pooling benefits of horizontal coordination in a newsvendor environment. In this example, we explore the benefits of vertical coordination in a supply chain consisting of a single retailer and a single supplier. One way that supply-chain members share risks is by having the supplier sell some goods on consignment, whereby the goods remain the supplier's property even though located at the retailer. Businesses with very uncertain demand, such as jewelers, may be particularly likely to promote consignment (Munson et al. 1999).

The newsvendor problem is often taught in introductory operations management classes. The problem arises when a retailer must make a one-time purchase of a single product to meet uncertain customer demand. For a simple one-level newsvendor problem, if we let O denote the overage cost per unit and U denote the underage cost per unit, the optimal order size Q^* is chosen such that $F(Q^*) = U/(O+U)$, where $F(x)$ is the cumulative distribution function of the random customer demand X (Evans 1997). However, the order quantity that maximizes profits for the retailer may not maximize the total supply-chain profits when we also consider the cost structure of the retailer's supplier. Example 6 illustrates how to coordinate lot sizes in a two-level newsvendor environment. We show that risk pooling via vertical coordination leads to higher order quantities (with a lower risk of unmet demand but a higher risk of ending with excess supply) and, more important, leads to higher profits for the channel.

We assume that the supplier has a lot-for-lot policy and will order and sell to the retailer the amount the retailer requests. Let P_s and P_r be the prices charged by the supplier and retailer, respectively. Let C_s be the supplier's manufacturing cost per unit, and let C_r be the retailer's cost per unit, exclusive of purchasing cost P_s. Let V be the salvage value of any unsold units at the end of the selling season. Let Q_c^* and Q_u^* be the optimal order size under a coordinated system and an uncoordinated system, respectively. (We derive the following results in the Appendix.)

If the retailer acts independently, the lot size should be chosen such that $F(Q_u^*) = (P_r - C_r - P_s)/(P_r - V)$. The lot size in a coordinated system should be chosen such that $F(Q_c^*) = (P_r - C_r - C_s)/(P_r - V)$. The (nonnegative) profit increase for the supply chain due to coordination is

$$(P_r - C_s - C_r)(Q_c^* - Q_u^*) - (P_r - V)\int_{Q_u^*}^{Q_c^*} F(x)\,dx.$$

The order size will be increased when there is coordination between the two firms because $C_s < P_s$. The supplier's profits increase with the joint optimization, but the retailer's profits decrease. Therefore, some of the increase in total channel profits should be redistributed to the retailer as an incentive for coordination through quantity discounts or some other method.

Typical classroom examples illustrate the basic newsvendor problem by using either the normal or the uniform distribution. We can use the following equation using the Excel commands *NORMSDIST* and *NORMDIST* to approximate the expected profit of ordering Q units when the demand is normally distributed with mean μ and standard deviation σ (modified from Chopra and Meindl 2001):

$$\pi = (U+O)\left[\mu \cdot NORMSDIST\left(\frac{Q-\mu}{\sigma}\right) - \sigma \cdot NORMDIST\left(\frac{Q-\mu}{\sigma}, 0, 1, 0\right)\right].$$

When the demand is uniformly distributed in the interval (a, b), the expected profit of ordering the optimal quantity equals

$$\pi = aU + \frac{(b-a)U^2}{2(U+O)}.$$

(We provide the derivation in the Appendix.)

For a numerical example of a normal distribution, assume that the market demand for the product follows a normal distribution with a mean of 1,000 units and a standard deviation of 500 units. In addition, $C_s = C_r = \$20$, $P_s = \$50$, $P_r = \$100$ (thus both the supplier and the retailer have the same \$30 profit margin), and $V = \$10$.

If the firms act independently, $U = 30$ and $O = 60$, so the retailer will order (using the Excel command *NORMINV*) $Q_u^* = NORMINV(30/(60+30), 1000, 500) = 785$ units. The retailer's expected profits are $(\$30+\$60)[1000 \cdot NORMSDIST((785-1000)/500) - 500 \cdot NORMDIST((785-1000)/500, 0, 1, 0)] = \$13{,}638$. The supplier's profits are $(\$50-\$20)785 = \$23{,}550$. Consequently, the total channel profits are $\$13{,}638 + \$23{,}550 = \$37{,}188$.

Alternatively, if the two firms are considered as one organization, $U = 60$ and $O = 30$, and the best order size is $Q_c^* = NORMINV(60/(30+60), 1000, 500) = 1{,}215$ units. Total channel profits are $(\$60+\$30)[1000 \cdot NORMSDIST((1215-1000)/500) - 500 \cdot NORMDIST((1215-1000)/500, 0, 1, 0)] = \$43{,}638$, which represents a 17.34 percent improvement over independent optimization.

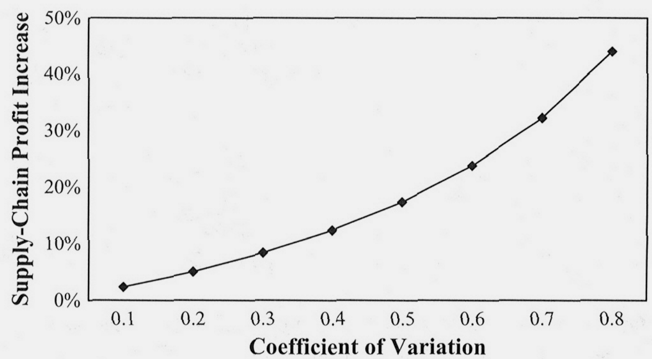

Figure 4: Sensitivity analysis on the numerical example for the coordinated newsvendor lot sizes with normally distributed demand shows that, when all the other parameters remain unchanged, the percent of profit increase becomes larger when the coefficient of variation increases (by increasing the standard deviation of demand). This suggests that the benefits from channel coordination are greater when the demand has more variability.

Sensitivity analysis performed on this example by altering the standard deviation (thus changing the coefficient of variation) suggests that the value of coordination increases as the uncertainty of demand increases (Figure 4).

Suppose that the cost factors remain unchanged, but demand is uniformly distributed between 5,000 and 15,000 units. Without coordination, the retailer will set its order quantity at $Q_u^* = 5{,}000 + (15{,}000 - 5{,}000)[30/(60+30)] = 8{,}333$ units. Its expected profits are $5{,}000(\$30) + [(15{,}000-5{,}000)(\$30^2)]/[2(\$30+\$60)] = \$200{,}000$. The supplier's profits equal $(\$50-\$20)8{,}333 = \$249{,}990$. Thus, the total channel profits are $\$200{,}000 + \$249{,}990 = \$449{,}990$.

Alternatively, if the two firms are considered as one organization, $Q_c^* = 5{,}000 + (15{,}000-5{,}000) \times [60/(30+60)] = 11{,}667$ units. Total channel profits are $5{,}000(\$60) + [(15{,}000-5{,}000)(\$60^2)]/[2(\$60+\$30)] = \$500{,}000$, representing an 11.11 percent improvement over independent optimization.

Conclusion

Cooperation between supply-chain members may be easier said than done. Chopra and Meindl (2001) describe major obstacles to supply-chain coordination falling into five categories: incentive obstacles, information-processing obstacles, operational obstacles, pricing obstacles, and behavioral obstacles. Taken to the extreme, our arguments here might imply that firms should simply vertically integrate to bypass certain obstacles and create the natural incentive to cooperate throughout the supply chain and thus to eliminate the inefficiencies that arise. However, most real-world companies do not vertically integrate to an extreme, which implies that strong barriers to vertical integration exist as well (Williamson 1985). Instead, forward-looking members of supply chains are finding innovative ways to create a spirit of cooperation.

A basic premise of supply-chain management is that communication and coordination can greatly enhance the effectiveness of the supply chain, creating financial benefits that the cooperating members of the chain can share. Mechanisms to encourage cooperation can take a variety of forms, including quantity discounts (Chopra and Meindl 2001). As with any group of entities, when all members effectively integrate their efforts, synergies may emerge. In supply chains in particular, the actions of rational managers of firms acting independently create natural inefficiencies that would not exist if the supply-chain members coordinated their efforts. Numerical examples can clearly illustrate these effects. We have collected examples suitable for classroom use that arise in common areas where companies use and abuse power: inventory control, pricing control, information control, control over the channel structure, and operations control (Munson et al. 1999). Similar examples could be developed to address other issues, such as transportation costs or joint advertising ventures. We hope that future managers will recognize that success in today's global marketplace demands close attention to all supply-chain functions and a constant search for ways to work with supply-chain partners to better compete together against other powerful supply chains.

Appendix

Proofs for Example 5: Coordinated Pricing with Multiple Supplier Tiers

The following propositions extend the coordinated pricing strategy to a supply chain with one retailer and $N-1$ single supplier tiers. Let π_c represent the system profit under coordinated pricing, and let π_u represent the system profit under uncoordinated pricing. Let C_i be the marginal cost of firm i ($i = 1, 2, \ldots, N$) where $i = 1$ denotes the retailer, $i = 2$ denotes the retailer's supplier, $i = 3$ denotes the retailer's supplier's supplier, and so forth. With the exception of firm N (the most upstream member of the supply chain), C_i does not include the purchase price. Let P_i denote the price charged by firm i. The decision variable Q represents the quantity sold to final customers, and Q^* represents the optimal (profit-maximizing) quantity. The retailer faces a deterministic linear demand curve of the form $P_1 = a - bQ$ ($a, b > 0$).

Proposition 1. *If there is coordination among the N firms,*

$$Q^* = \frac{1}{2b}\left(a - \sum_{i=1}^{N} C_i\right) \quad \textit{and} \quad \pi_c = \frac{1}{4b}\left(a - \sum_{i=1}^{N} C_i\right)^2.$$

Proof. If the N firms are considered as one organization, marginal revenue $= a - 2bQ$, and marginal cost $= \sum_i C_i$. Setting these equal yields the Q^* stated in the proposition. Plugging this into the retailer's demand function yields the retail price $P_1 = (a + \sum_i C_i)/2$. With no intercompany transactions, the system profit is $\pi_c = Q^*(P_1 - \sum_i C_i) = (a - \sum_i C_i)^2/(4b)$. $\square$

Proposition 2. *If there is no coordination among the N firms,*

$$Q^* = \frac{1}{2^N b}\left(a - \sum_{i=1}^{N} C_i\right)$$

and

$$\pi_u = (2^{2-N} - 2^{2-2N})\frac{1}{4b}\left(a - \sum_{i=1}^{N} C_i\right)^2.$$

PROOF. The tier 1 supplier ($i=2$) knows that the retailer will choose the quantity that equates its marginal revenue ($a-2bQ$) with its marginal cost (C_1+P_2). Solving for P_2 yields the derived demand curve facing the tier 1 supplier: $P_2=(a-C_1)-2bQ$. Continuing in this fashion up the supply chain,

$$P_m=\left(a-\sum_{i=1}^{m-1}C_i\right)-2^{m-1}bQ$$

for $m=1,2,\ldots,N$. The uppermost supplier, N, has a marginal revenue of

$$\left(a-\sum_{i=1}^{N-1}C_i\right)-2^{N}bQ$$

and a marginal cost of C_N. Equating these and solving for Q yields the Q^* stated in Proposition 2. The profit for firm m equals $Q^*(P_m-P_{m+1}-C_m)$ (where $P_{N+1}=0$), which reduces to

$$\frac{2^{m-2N+1}}{4b}\left(a-\sum_{i=1}^{N}C_i\right)^2.$$

Summing these over all N firms produces a geometric progression that reduces to the system profit stated in Proposition 2. □

PROPOSITION 3. *The system percentage profit increase from coordinated pricing vs. individual optimization is* $(2^{2N-2}-2^N+1)/(2^N-1)$.

PROOF. From Propositions 1 and 2, $(\pi_c-\pi_u)/\pi_u=[1-(2^{2-N}-2^{2-2N})]/(2^{2-N}-2^{2-2N})$. Multiplying the numerator and denominator by 2^{2N-2} yields the result stated in Proposition 3. □

Proofs for Example 6: Two-Level Newsvendor Problem

Lot Sizes. If the retailer acts independently, its underage and overage costs are $U_u=P_r-(C_r+P_s)$ and $O_u=(C_r+P_s)-V$, respectively. The ratio $U_u/(O_u+U_u)$ reduces to $[P_r-(C_r+P_s)]/(P_r-V)$. If the firms coordinate, the system underage and overage costs are $U_c=P_r-(C_r+C_s)$ and $O_c=(C_r+C_s)-V$, respectively. The ratio $U_c/(O_c+U_c)$ reduces to $[P_r-(C_r+C_s)]/(P_r-V)$. □

Expected Profit. Define $f(x)$ as the density function of random demand X. Under independent optimization, by adding the supplier's profit to Chopra and Meindl's (2001) expected profit function for the retailer, we obtain

$$\begin{aligned}\pi(Q_u^*)&=\int_0^{Q_u^*}[xU_u-(Q_u^*-x)O_u]f(x)\,dx\\&\quad+\int_{Q_u^*}^{\infty}Q_u^*U_uf(x)\,dx+(P_s-C_s)Q_u^*\\&=(P_r-C_s-C_r)Q_u^*-(P_r-V)\int_0^{Q_u^*}F(x)\,dx.\end{aligned}$$

Alternatively, if the two firms coordinate, the expected system profit becomes

$$\begin{aligned}\pi(Q_c^*)&=\int_0^{Q_c^*}[xU_c-(Q_c^*-x)O_c]f(x)\,dx\\&\quad+\int_{Q_c^*}^{\infty}Q_c^*U_cf(x)\,dx\\&=(P_r-C_s-C_r)Q_c^*-(P_r-V)\int_0^{Q_c^*}F(x)\,dx.\end{aligned}$$

The profit change, $\Delta\pi$, due to coordination is $\pi(Q_c^*)-\pi(Q_u^*)$, which reduces to

$$(P_r-C_s-C_r)(Q_c^*-Q_u^*)-(P_r-V)\int_{Q_u^*}^{Q_c^*}F(x)\,dx.$$

To show that $\Delta\pi$ is nonnegative, we utilize the nondecreasing property of $F(x)$, that is

$$\int_{Q_u^*}^{Q_c^*}F(x)\,dx\le(Q_c^*-Q_u^*)F(Q_c^*)=\frac{(Q_c^*-Q_u^*)(P_r-C_s-C_r)}{P_r-V}.$$

Therefore,

$$\begin{aligned}\Delta\pi&\ge(P_r-C_s-C_r)(Q_c^*-Q_u^*)\\&\quad-[(P_r-V)(Q_c^*-Q_u^*)(P_r-C_s-C_r)/(P_r-V)]\\&=0.\end{aligned}$$ □

Expected Profit for the Uniform Distribution. If demand is uniformly distributed between a and b, the expected profit of ordering Q units is

$$\begin{aligned}\pi(Q)&=\int_a^Q[xU-(Q-x)O]f(x)\,dx+\int_Q^bQUf(x)\,dx\\&=QU-(U+O)\int_a^QF(x)\,dx\\&=QU-(U+O)\int_a^Q\frac{x-a}{b-a}\,dx.\end{aligned}$$

The optimal Q^* for the uniform distribution is $a + (b-a)[U/(O+U)]$; therefore,

$$\pi(Q^*) = U\left[a + \frac{(b-a)U}{U+O}\right] - \frac{U+O}{b-a}\left\{\frac{1}{2}\left[a + \frac{(b-a)U}{(U+O)}\right]^2 - a\left[a + \frac{(b-a)U}{U+O}\right] + \frac{1}{2}a^2\right\}$$

$$= aU + \frac{(b-a)U^2}{2(U+O)}. \quad \square$$

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